Anisentropic Gas Flow

C N KAUL*
Indian Institute of Technology, Kharagpur, India

For one-dimensional plane, cylindrical, or spherical-symmetric anisentropic gas flow, relations between the rates of propagation of constant pressure, density, and temperature are established

1 Introduction

A DECAYING shock leaves behind an anisentropic gas flow, i e , one in which the specific entropy, though remaining constant for a particular fluid particle, varies from particle to particle The anisentropic flow of a perfect polytropic gas, devoid of viscosity and heat conductivity, and depending upon one space coordinate r, the distance measured from a fixed center, and upon time t, is governed by the following equations[1]:

$$\rho_t + \rho\, u + \rho u \;+ (\sigma\rho u/r) = 0 \quad (1)$$

$$\rho(u_t + uu\;) + p \;= 0 \quad (2)$$

$$S_t + uS \;= 0 \quad (3)$$

$$p = \exp(s - s_0/c_p)\rho^\gamma \quad (4)$$

where the various symbols have their usual meaning The constant $\sigma = 0,1,2$ for the plane, cylindrical, or spherical-symmetric case Suffixes in the preceding equations and in sequel denote partial differentiation

2 Rates of Propagation of Constant Pressure and Density

Equation (3), in conjunction with (1) and (4), reduces to

$$\rho(p_t + up\;) + \gamma p[\rho u_r + (\sigma\rho u/r)] = 0 \quad (5)$$

In the r, t plane, the slope of isobars shall be given by

$$(dr/dt)_{p=\text{const}} = -(p_t/p_r)$$

which, with (2) and (5), yields

$$(dr/dt)_{p=\text{con t}} = u + (\gamma p/p\;)[u \;+ (\sigma u/r)]$$

If

$$R_p = (\gamma p/p\;)[u \;+ (\sigma u/r)] \quad (6)$$

then R_p, so defined, represents the rate at which constant pressure is propagated with respect to the gas In a similar manner R_ρ, the rate at which constant density is propagated with respect to the gas, can be evaluated with the help of Eq (1) to be

$$R_\rho = (\rho/\rho\;)[u \;+ (\sigma u/r)] \quad (7)$$

From (6) and (7) we have

$$R_p p\;/R_\rho \rho \;= \gamma p/\rho = c^2 \quad (8)$$

where c is the local speed of sound For an isentropic flow $p \;= c^2\rho$, and then from (8), $R_p = R_\rho$ as it should From (8) it follows that for a polytropic gas the local speed of sound is the geometric mean of the ratios R_p/R_ρ and $p\;/\rho$

3 Rate of Propagation of Absolute Temperature

For a perfect gas, $p = \rho RT$, where T is the absolute temperature and R the gas constant Accordingly, the slope of the isothermals in the r,t plane shall be given by

$$\left(\frac{dx}{dt}\right)_{T=\text{ onst}} = \left(\frac{\rho p_t - \rho_t p}{p\rho \;- \rho p}\right)$$

This, with (1) and (5), yields R_T, the rate at which absolute temperature is propagated with respect to the gas, as

$$R_T = \frac{(\gamma - 1)p\rho[u_r + (\sigma u/r)]}{(\rho p \;- p\rho\;)} \quad (9)$$

Equations (6–8) then give

$$(\gamma - 1)/R_T = \gamma/R_p - 1/R_\rho \quad (10)$$

a result first obtained by Ludford and Martin,[2] for the case $\sigma = 0$, from the formulation of plane ($\sigma = 0$) one-dimensional anisentropic gas flow as given by Martin [3] From (10), we have the result that for a plane, cylindrical, or spherical-symmetric anisentropic gas flow, the rate at which the absolute temperature is propagated with respect to the gas is a weighted harmonic mean of the rates at which density and pressure are propagated with respect to the gas

References

[1] Courant, R and Friedrichs, K O , *Supersonic Flow and Shock Waves* (Interscience Publishers, Inc , New York, 1948), p 28

[2] Ludford, G S S and Martin, M H , "One-dimensional anisentropic flows," Commun Pure Appl **VII**, 45–63 (1954)

[3] Martin, M H "The propagation of a plane shock into a quiet atmosphere, Can J Math **5**, 37–39 (1953)

Received December 27, 1963

* Department of Mathematics

Modification of Weierstrass-Erdmann Corner Conditions in Space Navigations

R N BHATTACHARYA* AND M BHATTACHARJEE†
Jadavpur University, Calcutta, India

Mayer's problem is of wide application in the optimization studies of space trajectories One of the two corner conditions due to Erdmann and Weierstrass gives the continuity of the partial derivatives of the Lagrange function with respect to such time derivatives of the space coordinates as are discontinuous at the corner If some of the space coordinates themselves are discontinuous there, then the partial derivatives of the Lagrange function, with respect to the time derivatives of the corresponding coordinates, must vanish on either side of the corner, in addition to being continuous This has been investigated and established in the following note A nontrivial application of this modified corner condition is under preparation and may be published soon

1 Introduction

MAYER'S problem in the calculus of variations provides an important tool for the determination of optimal trajectories of space vehicles An abstract theoretical formulation of the problem is given by Bliss [1] Lawden[2] has given a simplified formulation of the problem which is more suitable for the application to space dynamics Weierstrass-Erdmann corner conditions are derived from the concept of a corner as a point of discontinuity of the time derivatives of the space coordinates of the rocket The space coordinates themselves are treated as continuous at the corners The essential requirement of the continuity of a dynamical trajectory involves the idea of continuity of its cartesian space coordinates But if we use polar coordinates, or other forms of the generalized coordinates, we get discontinuities of space coordinates such as the vectorial angle θ without violating the essential requirement of the continuity of the trajectory

Received February 25 1964

* Reader Department of Mathematics

† Research Worker, Department of Mathematics